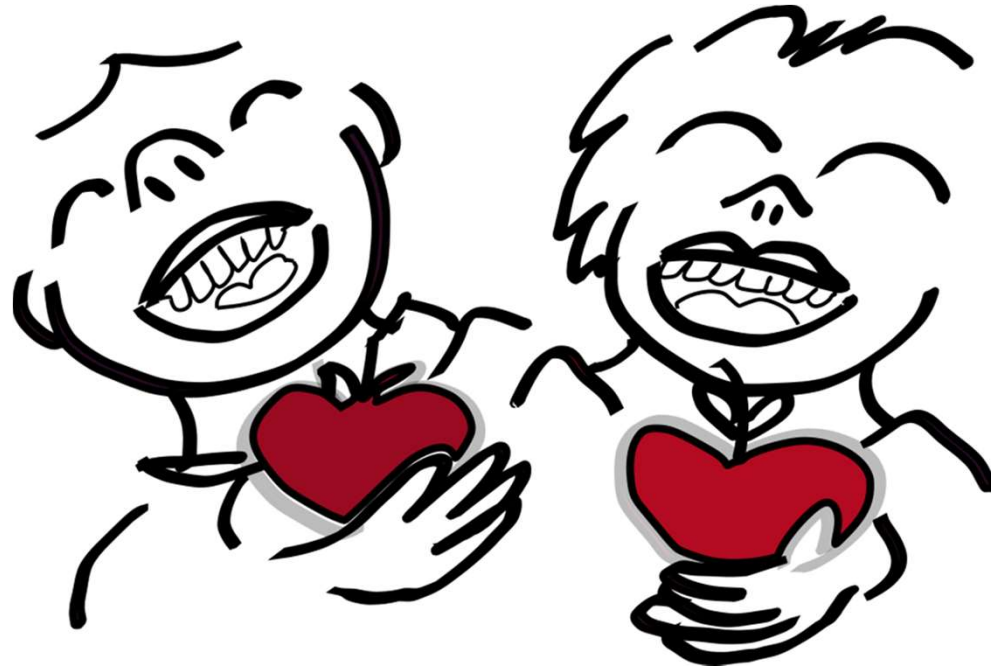


What if $1+1 \neq 2$?



Text by Lixin Luo

Illustrations by Abram Hindle & Lixin Luo

To my parents

-- L. L.

To mom and dad

-- A. H.

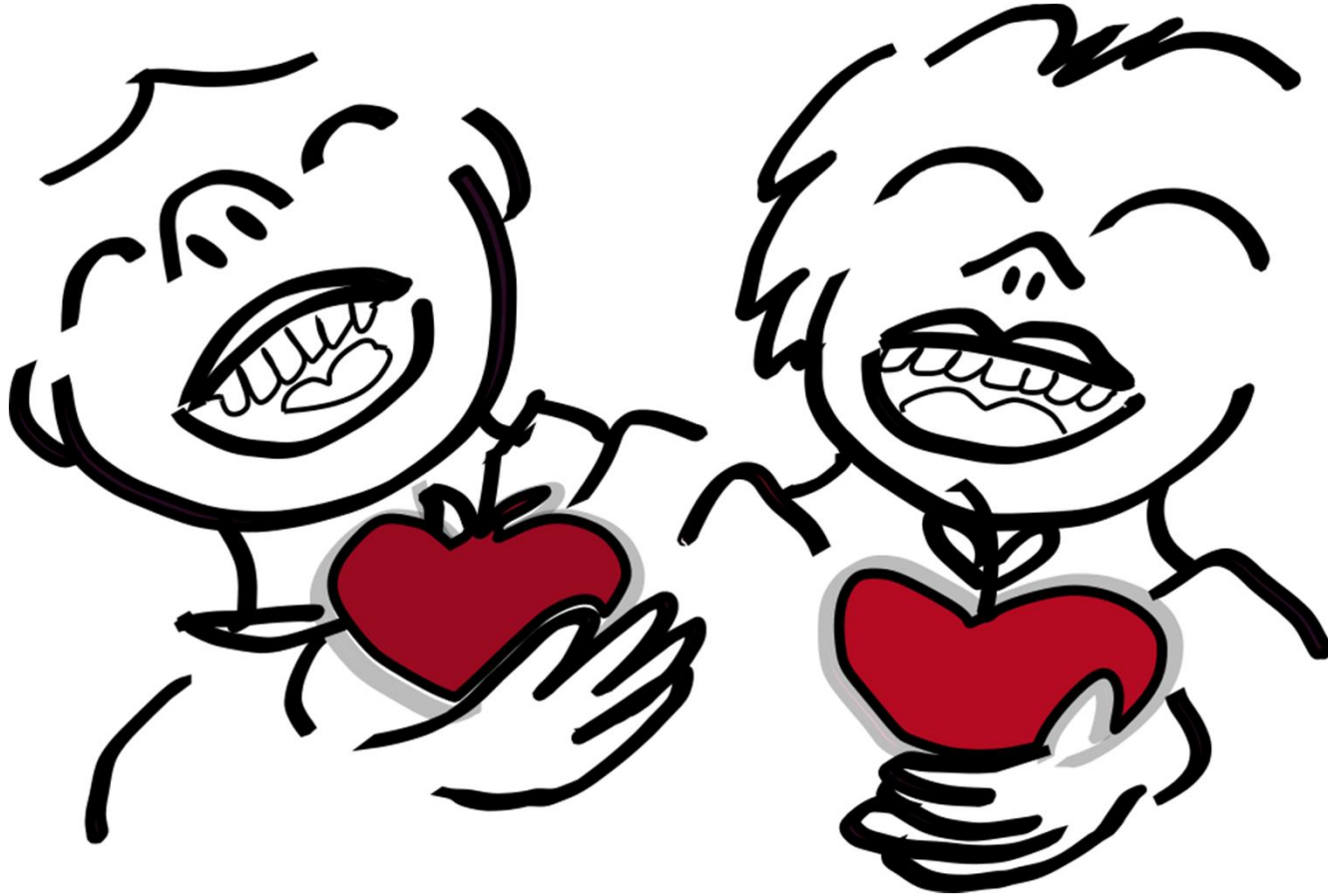


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For collaboration please write to lixinplayground@gmail.com

Made in Edmonton, AB, Canada

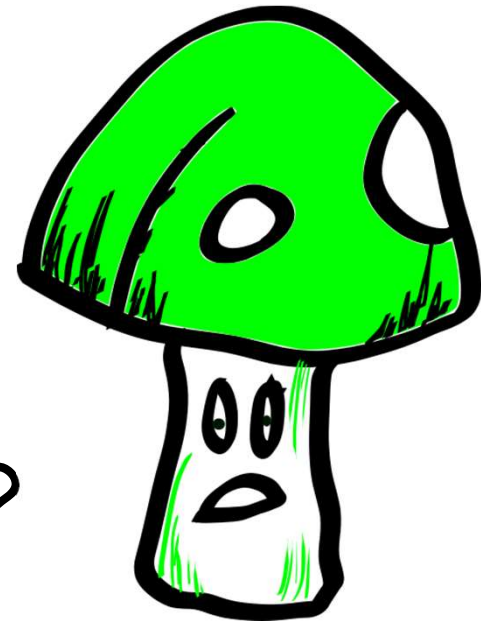


So we know $1+1=2$,
like when I get
an apple from dad and another from mom,
I have two apples.

What if $1+1 \neq 2$?



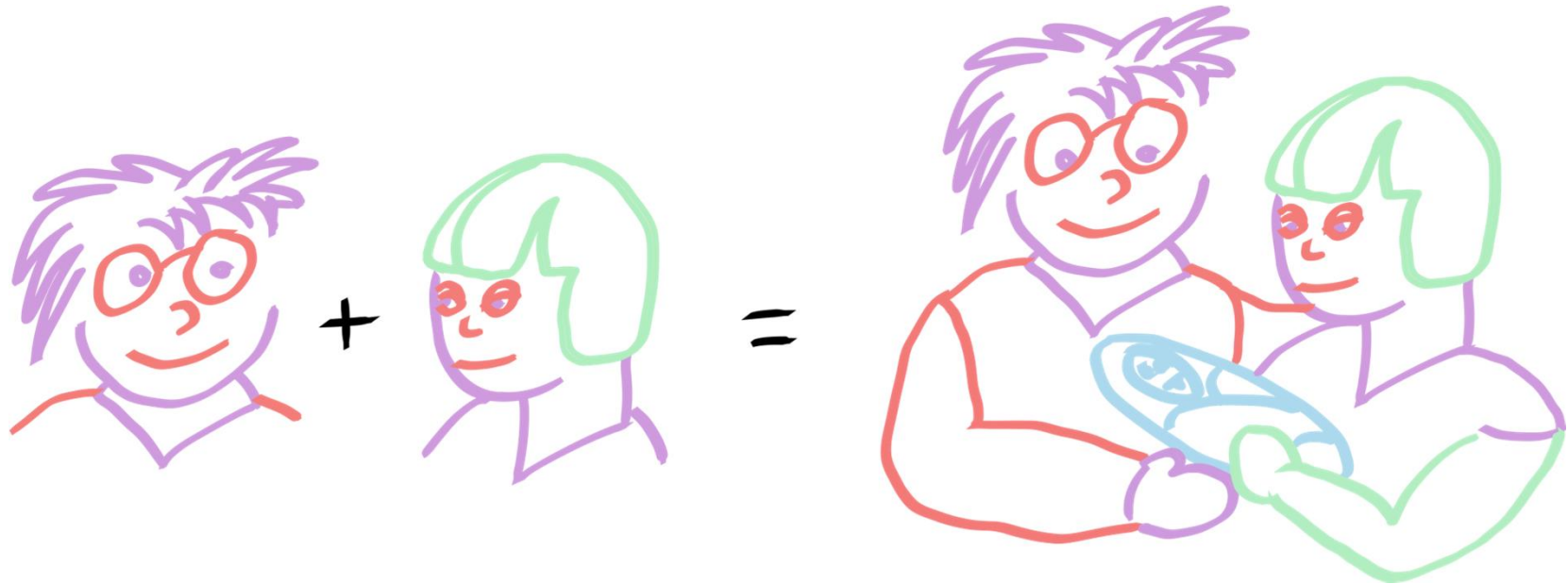
Then $1+1$ could be 3.



But... how?

1+1=3 is possible when

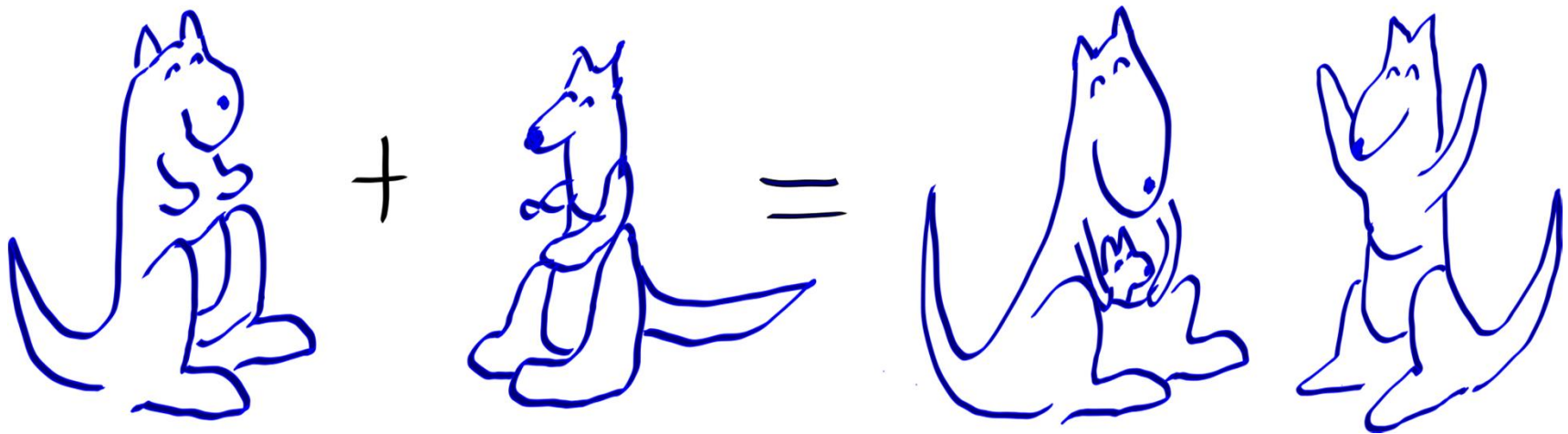
one person and one person together
make a family of three:



1 person + 1 person = 3 persons

1+1=3 is possible when

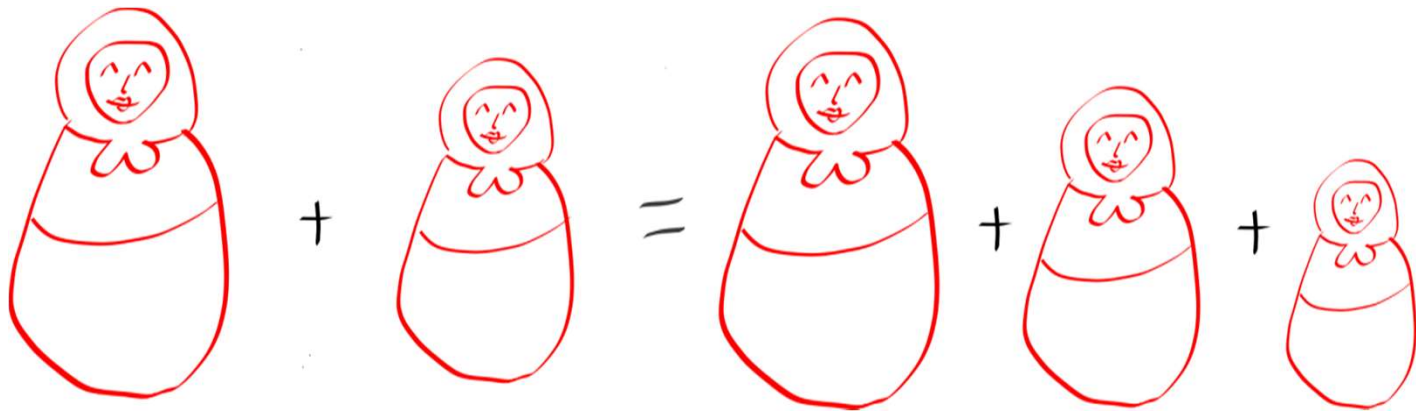
one kangaroo and another kangaroo are visiting and suddenly a joey pops out:



1 kangaroo + 1 kangaroo = 3 kangaroos

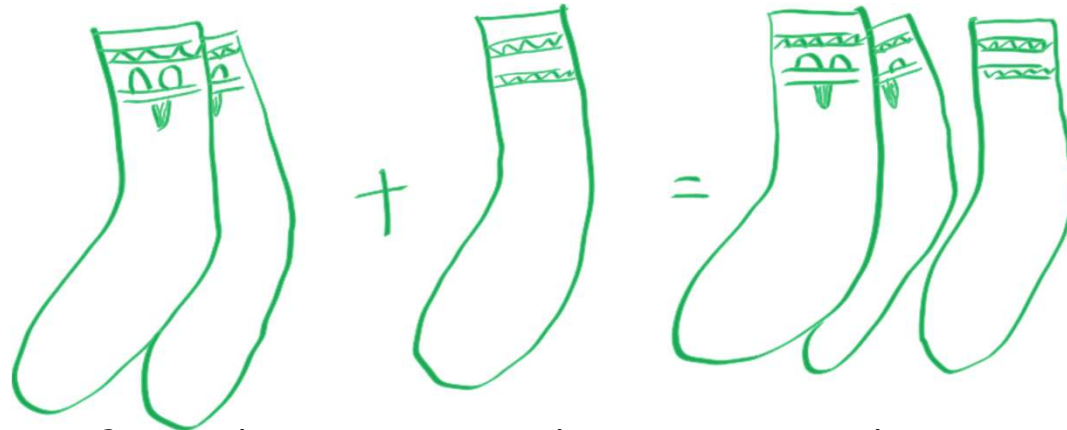
1+1=3 is possible when

one Russian doll and another one together
include three Russian dolls:

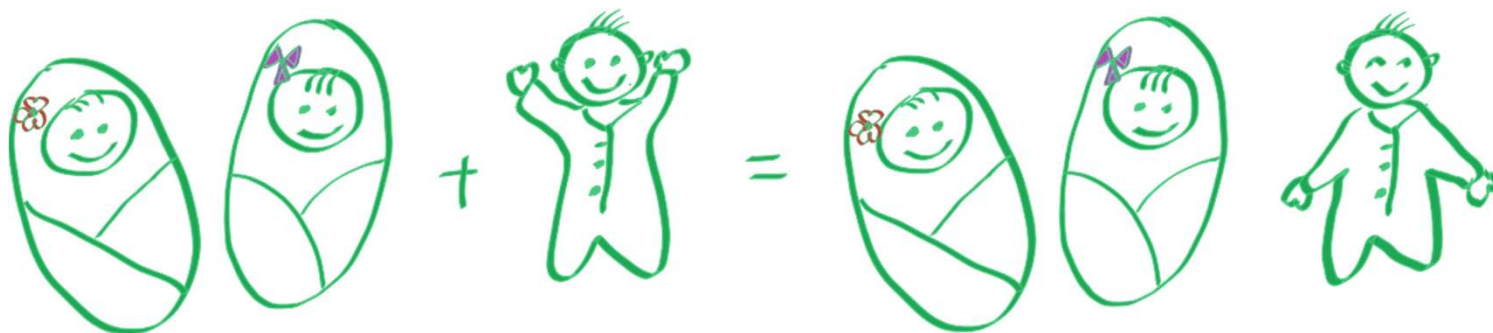


1 Russian doll + 1 Russian doll = 3 Russian dolls

$1+1=3$ is possible, also when



1 pair of socks + 1 sock = 3 socks



1 twin + 1 single = 3 people

Hmm, looks like
1 means different kinds of things



1



A thing

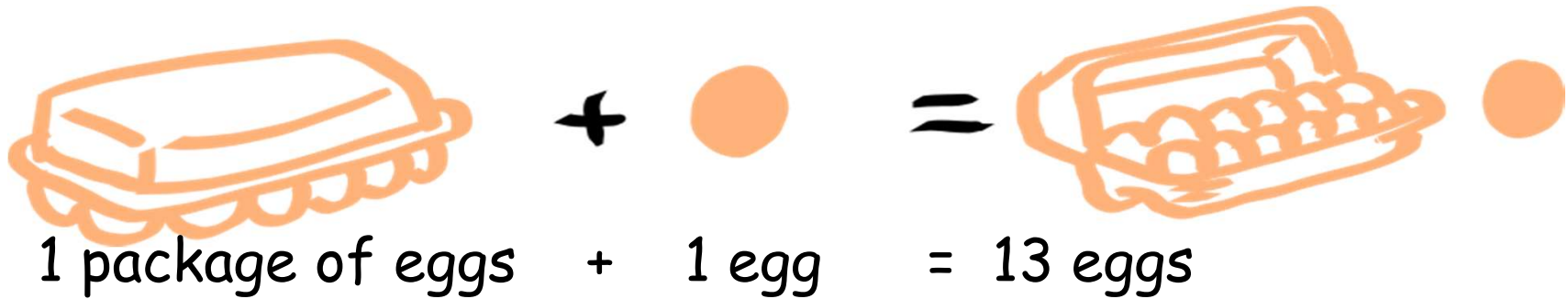


A group of things

If so, then $1+1 \neq 3$ is also possible, like:



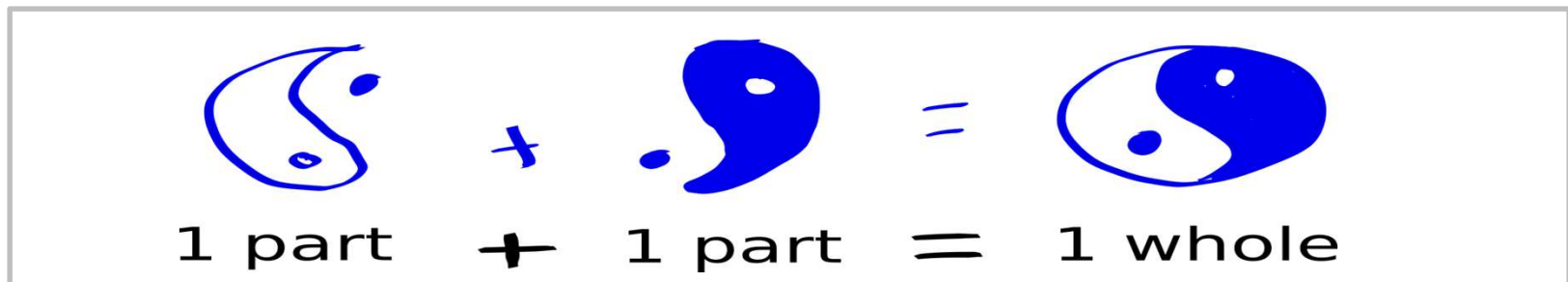
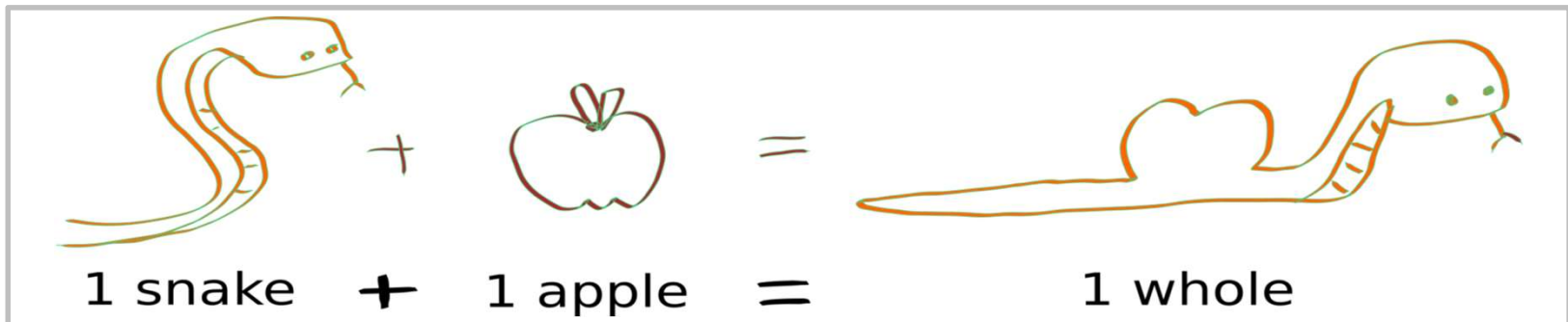
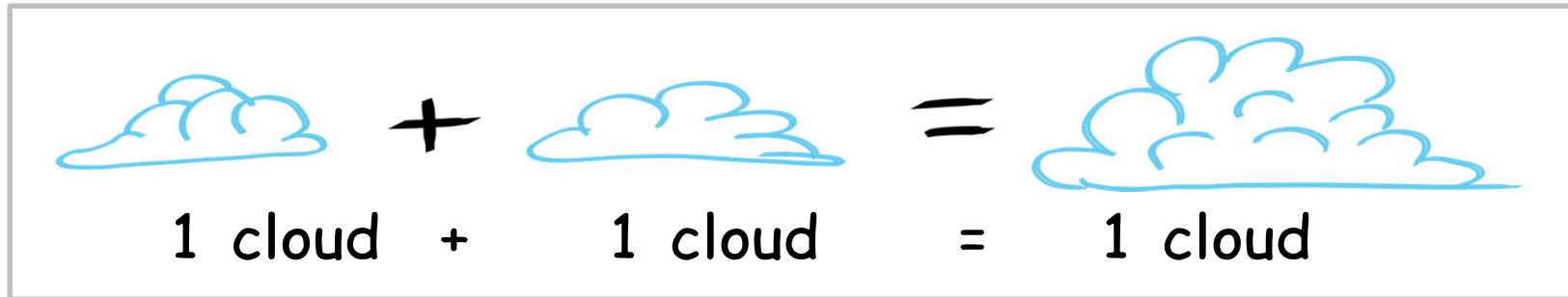
1 bag of apples + 1 bag of oranges = 39 fruit



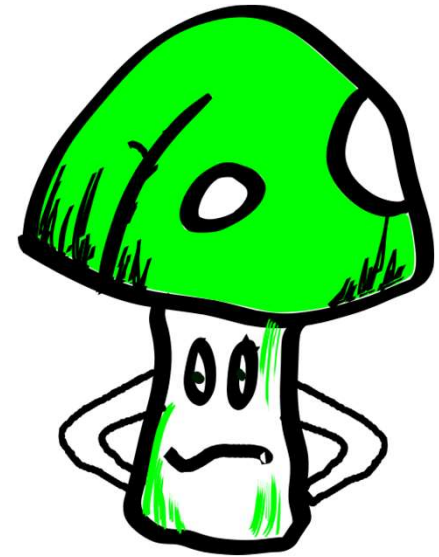
1 package of eggs + 1 egg = 13 eggs

1 kilometer + 1 meter = 1.001 kilometers or 1001 meters

We can even have many ways of getting 1, like $1+1=1$, as in



Wait!
This is getting out of control...



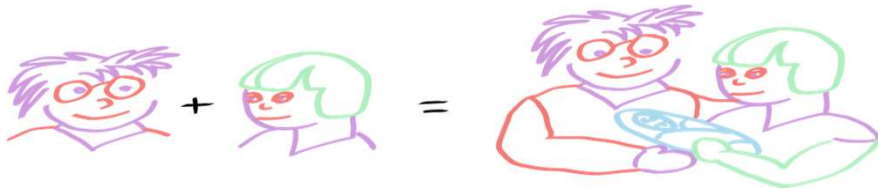
the symbol "1" is representing different types of things
"+" seems to mean more than adding
"=" seems to mean more than equal...



Let's go back
and
take a look at the
equations mentioned
earlier ...

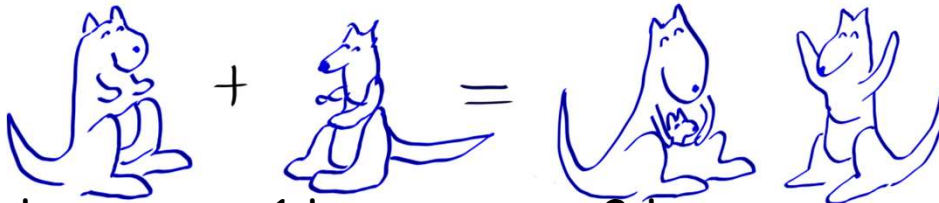
On pages 5-7, we count

We are actually counting:



1 person + 1 person = 3 persons

1 person + 1 person = 3 family members



1 kangaroo + 1 kangaroo = 3 kangaroos

1 adult kangaroo + 1 adult kangaroo = 3 kangaroos



1 Russian doll + 1 Russian doll = 3 Russian dolls

1 Russian doll + 1 nested Russian doll = 3 Russian dolls

So we count **twice**: before and after a change.

1 person + 1 person = 3 family members

BEFORE getting a baby

AFTER getting a baby

1 adult kangaroo + 1 adult kangaroo = 3 kangaroos

BEFORE a joey popped out

AFTER a joey popped out

1 Russian doll + 1 nested Russian doll = 3 Russian dolls

BEFORE a doll opened

AFTER a doll opened

Here we can understand

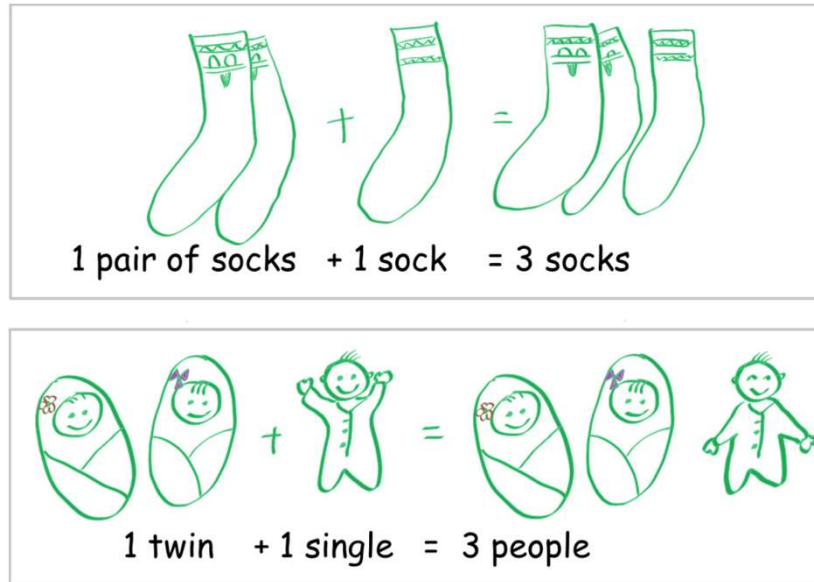
"=" as "changing into," and

"+" as "joining under a certain condition"
instead of "counting more of the same thing"

This is like a magician's work



On page 8, we count



There is no time change.

But the numbers don't always share the same unit either.

So "+" means "counting willfully" here

as we might change the units of the numbers or how we count.



Counting willfully also leads us to the equations on page 10:



1 bag of apples + 1 bag of oranges = 39 fruit



1 package of eggs + 1 egg = 13 eggs

1 kilometer + 1 meter = 1.001 kilometers or 1001 meters

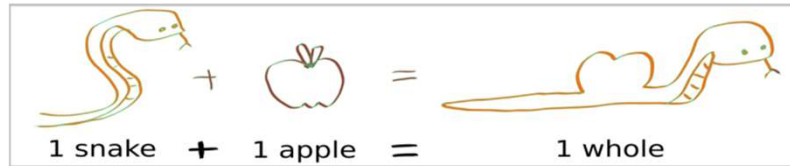
Here we **count inconsistently, using different units.**

For example, in the fruit equation,
the left side shows that we count *bags* of fruits but
the right side shows that we count the *number* of fruits.
We end up counting twice: one way then another.

Then I need to always remember what I
mean by each number. So much work!



On page 11, we count



Counting gets **more consistent**: In each equation,

The "1"s on the left represent a *part* and
the "1" on the right represents a *whole*.

But "+" means "**combining**" instead of "counting more".
So all equations represent parts united as a whole.

Well then, I can have $1+1+1+ \dots +1 = 1$ to show that
I can combine any numbers of parts into a whole.
Sounds deep? Or not?



If we try to be more consistent...



If we let

the numbers on each side of the equal sign
represent quantities with the same unit,
and let

"+" mean "adding the quantities",
then $1+1=3$ is possible when it represents
a conversion between quantities in two different units.
That is, 1 unit A + 1 unit A = 3 unit B.
Here "=" means "converting to".

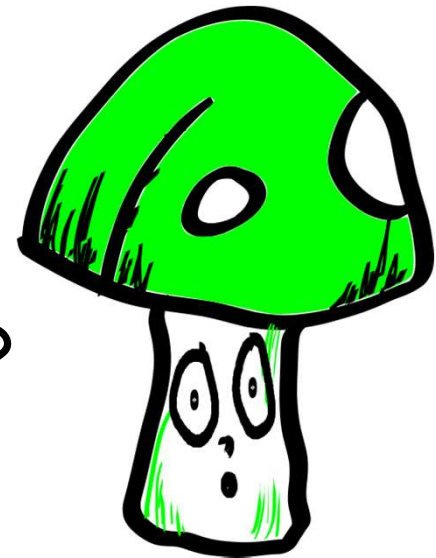
I get this. What you did here is the same like
 $1+1=120$ as in 1 hour + 1 hour = 120 minutes
 $1+1=4$ as in 1 two + 1 two = 4 ones



Using the same logic, $1+1 = 10$ is possible.



How?



1+1 = 10 is possible ...

if it represents

a conversion between base 10 and base 2 numbers.

That is, (1+1) in base 10 = 10 in base 2.

Note: "10" in base 2 is read as "one zero".

Base 10	Base 2
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111

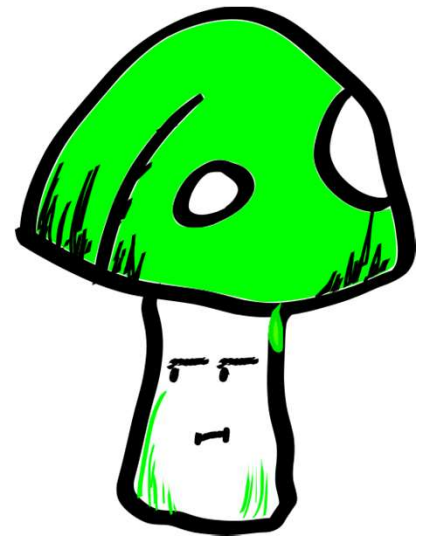
Base 2?
Is there base 3 then?



Now, for something quite different...



What if "+" does not mean
"adding quantities"?



Say, if "+" means connecting different texts,
then it is possible that $1+1 = 11$.

This is like how we create the word "sunflower" out of
"sun" and "flower": $\text{sun} + \text{flower} = \text{sunflower}$.

Here "1" and "11" are treated as strings or texts
instead of numbers.

Consequently, we could have

$$1+2 = 12,$$

$$123+45 = 12345,$$

$$a+b = ab,$$

and many more.



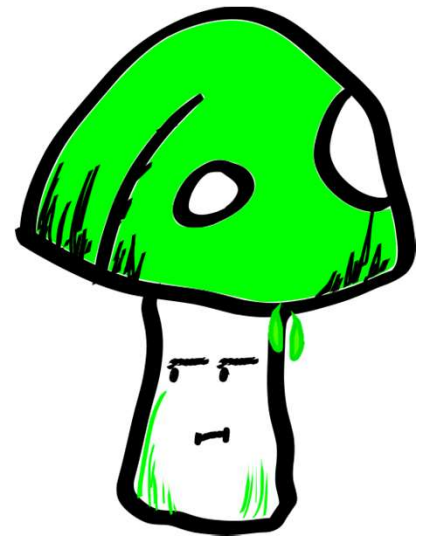
Wow, this is crazy!
Does this work for
banks?

This operation is used commonly in computer science:
It joins two strings together to make a new one.

Now, for something quite different, again!



What if "=" does not mean
"equal to"?



If we let

"=" mean "resembling"

then we could have

$$1+1 = 2+2 = a+a = a\text{⌘}a,$$

where "⌘" means an operation.

This says that

the expression of $1+1$ LOOKS like $2+2$,
or any number adding to itself,
or generally any text in operation with itself.

Consequently, we could have

$$1+2+1 = 2+1+2 = a+b+a$$

$$11+22 = 33+44 = aa+bb$$

$$1+1 = 1-1 = 1*1 = 1/1 = 1^1$$

and many more.

What?! It's all about the look now?
Does "=" mean "similar to" here?



Now, for something quite different, again!!



What if the whole equation is
an analogy?

I have a bad feeling
about this . . .



If we let

an equation represent an analogy

then we could have

$$1+1 = -1.$$

This can represent situations in which getting more becomes undesirable, such as

when adding more friends make us feel less connected,

when getting more stuff make us feel less fulfilled.

when having more choices make us less happy,

Consequently, we could have

$T = M$ representing time is like money,
and many more.

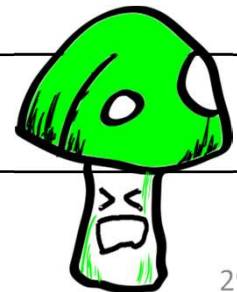
This is wild!
Are we still talking about math?



Now looking back our journey:
If $1+1 \neq 2$, then we might have

Possible Equations	Possible Conditions (When ...)
$1+1=3$	numbers don't share the same unit, or "=" means "changing into"
$1+1=1$	"=" means "combining"
$(1+1) \text{ unitA} = 3 \text{ unitB}$	unit conversion
$(1+1) \text{ in base 10} =$ 10 in base 2	number conversion in different base systems
$1+1=11$	"+" means joining texts
$1+1=2+2$	"=" means resembling
$1+1=-1$	equation represents analogy

and many more.



What if $1+1 = 0$,
what might this mean?
when might this be possible?



This book is a thought experiment of engendering recursive curricula starting from "What if $1+1 \neq 2$?" and see how far we can go by following "if...then..." logic and through repetition with variations. It is designed to

- invite revising the same idea and playing with equivalent forms of the same idea and equivalent ideas.
- encourage one to think less about equal but more about equivalent, and less about "what ___ is" but more about "what ___ might be" and "what ___ might be like".
- introduce learners to semiosis. It invites learners to consider mathematics language as signs that require interpretations. It aims to help learners to understand that the meanings of mathematical signs are conditional instead of universal or fixed.